

Study designs and statistical methods for current observational studies


Multilevel Analysis

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Study designs and statistical analysis in current observational studies

1. Difference-in-Differences
 2. Instrumental Variable Analysis
 3. Regression Discontinuity
 4. Panel Data Analysis / Interrupted Time-Series
 5. Propensity Score Analysis (matching, weighting, and adjustment using propensity scores)
 6. Adjustment(regression), weighting, stratification, and matching
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- Natural experimental methods (Quasi-experimental methods)

Multilevel Analysis

Analysis for the study population with the hierarchical structure and nested structure

Fig. Example of hierarchical structure

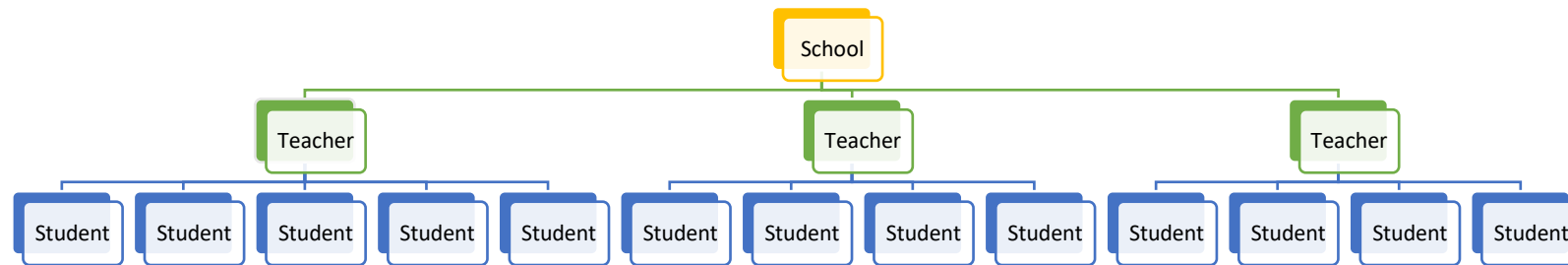


Table. Example of nested structure

id	week	weight
1	1	24
1	2	32
1	3	39
1	4	42.5
1	5	48
1	6	54.5
1	7	61
1	8	65
1	9	72
2	1	22.5
2	2	30.5
2	3	40.5
2	4	45
2	5	51
:	:	:

Student observations within a teacher's classroom may be correlated.

Same as the hierarchical model

Recap: Ordinary Least Square (OLS)

$$y = \beta_0 + \beta_1 x + \mu$$

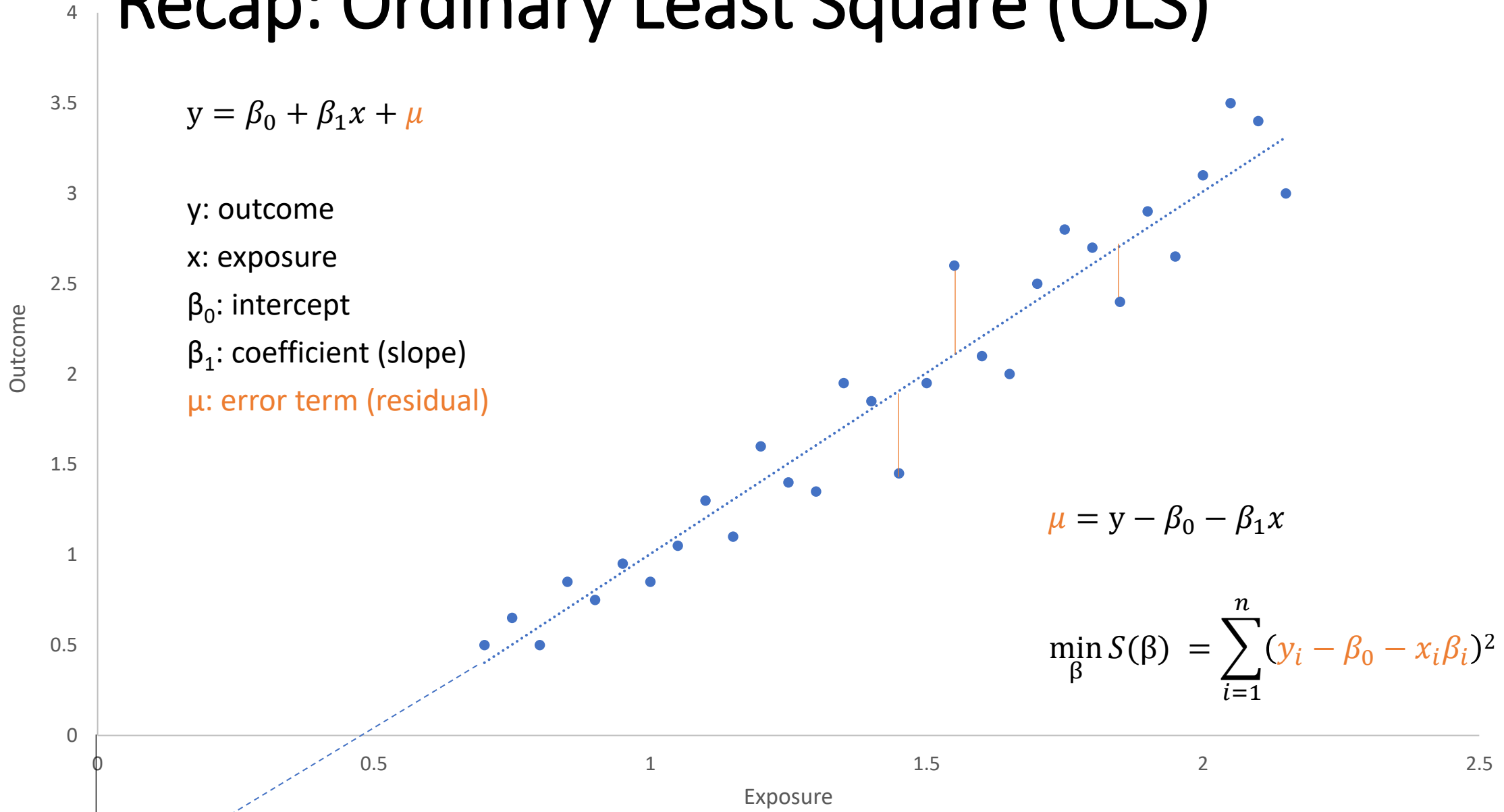
y: outcome

x: exposure

β_0 : intercept

β_1 : coefficient (slope)

μ : error term (residual)



$$\mu = y - \beta_0 - \beta_1 x$$

$$\min_{\beta} S(\beta) = \sum_{i=1}^n (y_i - \beta_0 - x_i \beta_1)^2$$

Gauss-Markov Assumptions

Best Linear Unbiased Estimator (BLUE)
: Efficient, Linear, Unbiased Estimator

Assumption SLR.1 (Linear in Parameters)

In the population model, the dependent variable, y , is related to the independent variable, x , and the error (or disturbance), u , as

$$y = \beta_0 + \beta_1 x + u,$$

where β_0 and β_1 are the population intercept and slope parameters, respectively.

Assumption SLR.2 (Random Sampling)

We have a random sample of size n , $\{(x_i, y_i) : i = 1, 2, \dots, n\}$, following the population model in Assumption SLR.1.

Assumption SLR.3 (Sample Variation in the Explanatory Variable)

The sample outcomes on x , namely, $\{x_i, i = 1, \dots, n\}$, are not all the same value.

Assumption SLR.4 (Zero Conditional Mean)

The error u has an expected value of zero given any value of the explanatory variable. In other words,

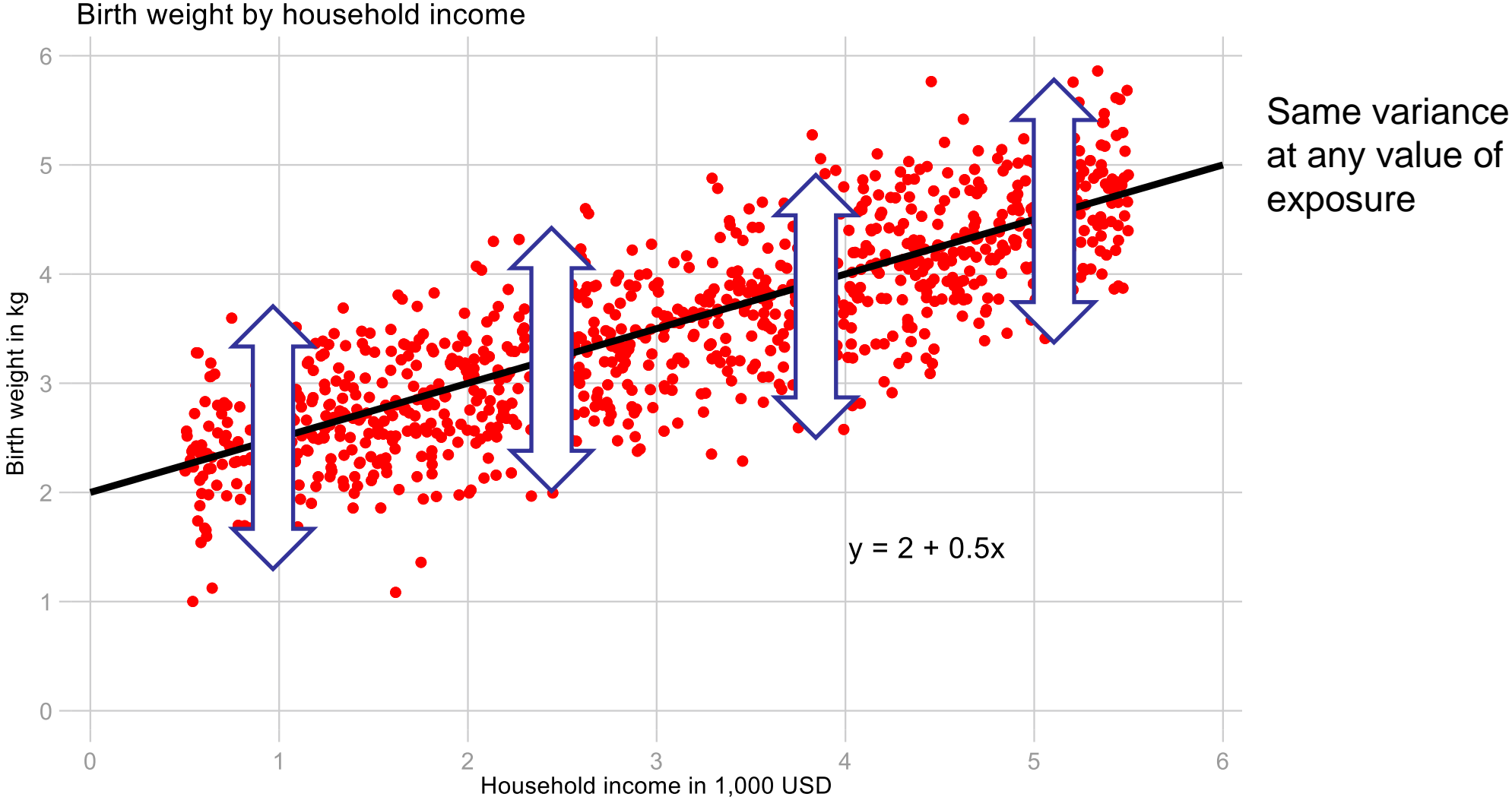
$$E(u|x) = 0.$$

Assumption SLR.5 (Homoskedasticity)

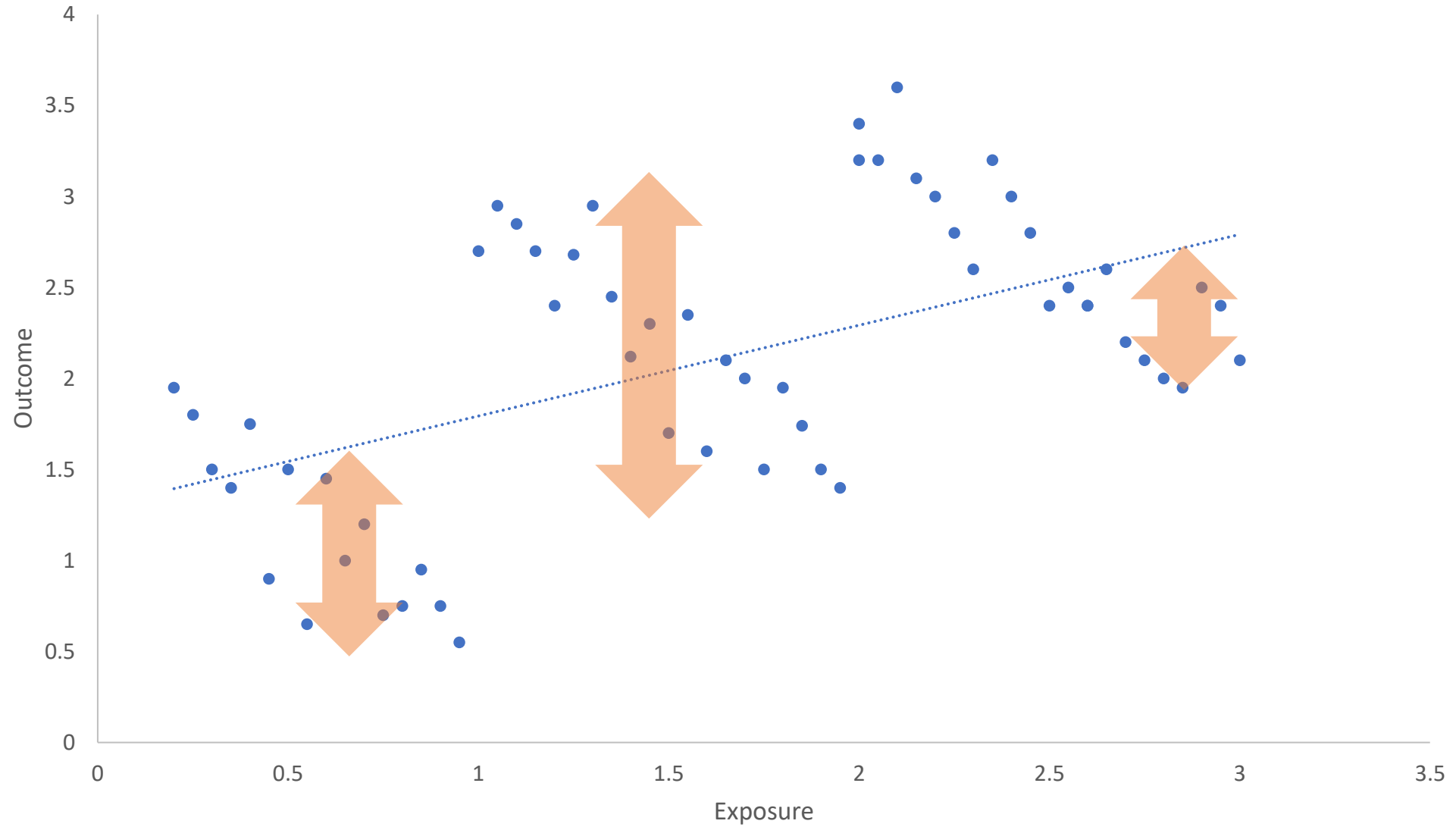
The error u has the same variance given any value of the explanatory variable. In other words,

$$\text{Var}(u|x) = \sigma^2.$$

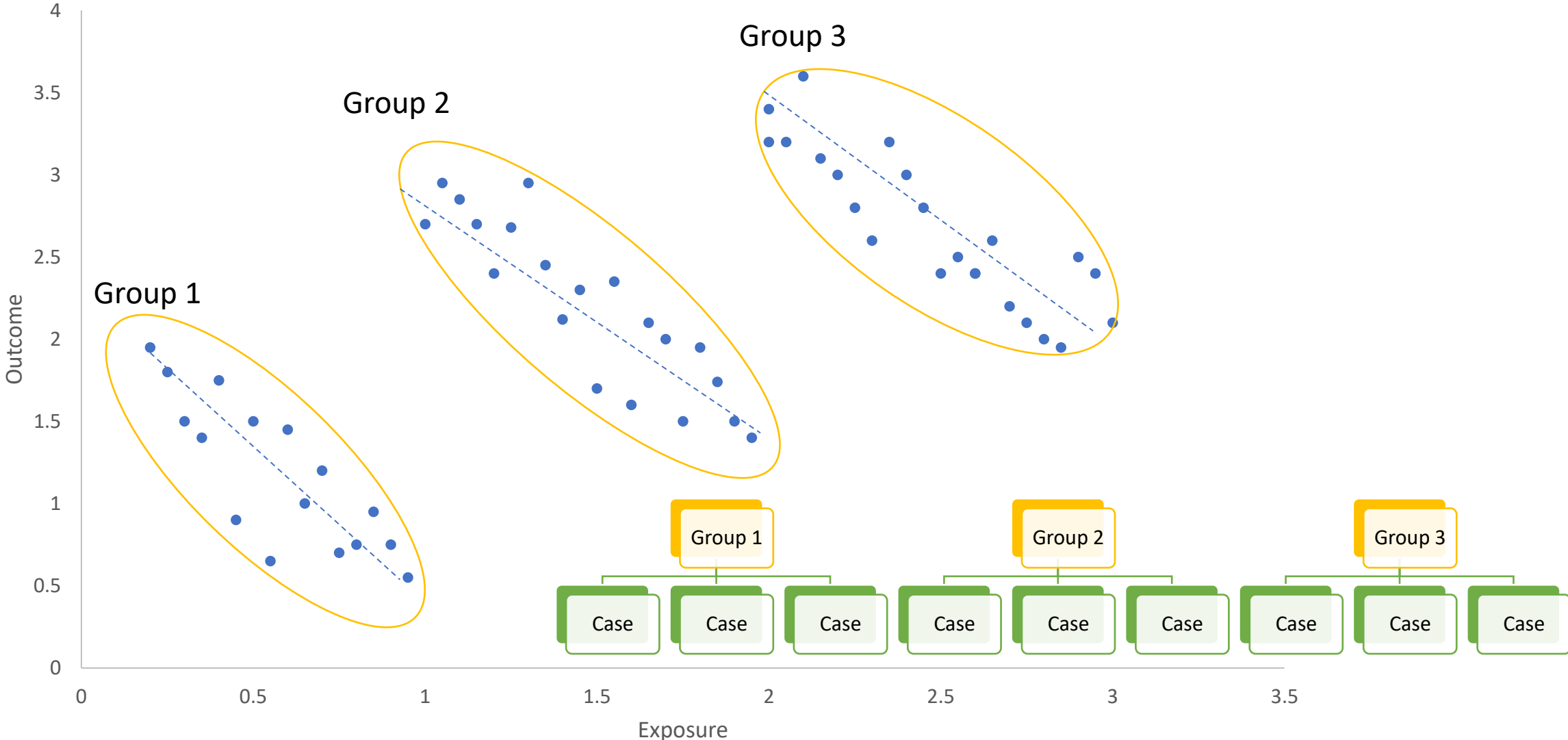
Homoskedasticity



Heteroskedasticity



Multilevel data



Continued only for students